Automated Design Optimization of Supersonic Airplane Wing Structures under Dynamic Constraints

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Theme

THE feasibility of performing optimal wing structure designs at a preliminary design stage was studied, and both an equivalent plate formulation and a finite element formulation were found to be useful. Dynamic constraints assumed important roles in the design procedure. The study focuses on the preliminary design stage and concerns the design of supersonic aircraft wings. It takes into account requirements on natural frequencies and flutter speed in addition to those on static responses and geometric side constraints. It includes an examination of the consequences of changing the aerodynamic envelope as well as the structural contents of the wing.

Contents

Structural design for minimum weight subject to constraints on natural frequencies were reported by Turner,[1] Zarghamee,[2] Rubin,[3] Fox and Kapoor,[4] et al. Schmit and Thornton[5] presented a rectangular supersonic wing of symmetric double wedge profile. Turner, in Ref. 6, extended his previous work[1] to the problem of attaining the minimum structural mass holding flutter speed constant. Recently, Stroud et al.[7] published work which is closely related to the work of this report.

The process involved in this study consists of two phases. The first is concerned with the determination of a simplified configuration of wing structure and the aerodynamic envelope. The simplified plan view shape of a low aspect ratio wing, the symmetric wing depth distribution and the cover panel thickness distribution are to be determined. An objective function composed of the aerodynamic drag and the weight of the wing is minimized, subject to constraints on static and dynamic behavior for multiple flight and fuel conditions. The second phase is the determination of relatively detailed material distribution for a wing of fixed configuration. Here the wing is modeled with finite elements and the thicknesses of the cover plate elements, cross-sectional areas of bar elements representing flanges, thickness of the webs, of ribs and spars, and the magnitude of "tuning" masses are determined for minimum weight, subject to static and dynamic constraints for multiple flight conditions.

The two phases are independent although it is possible to accomplish an extensive preliminary design of a wing by using the results of these two studies sequentially. Namely, first determine the basic configuration and crude material distribution by using the routine developed in Phase I. Then using the optimal configuration, detailed material distribution is obtained by the Phase II routine.

Formulation: The wing of a supersonic aircraft may be approximated as a chamberless, smooth, thin sandwich plate of variable thickness, and hence it is possible to consider this type of structure (equivalent plate model) as a plate which has variable flexural rigidities and shear stiffness. In Phase I detailed structural arrangement and dimensions are not considered. Inhomogenities of the core or the cover panels are smeared; i.e., the core is considered to be of a homogeneous, orthotropic material whose equivalent tensile and shear moduli are computed by smearing the material by the ratio of cross sectional areas, and the cover plates are also considered to be orthotropic plates of smooth thickness variation.

The wing planform is idealized as a trapezoid defined completely by four parameters: R, root; S, span; θ_1, leading edge; and θ_2, trailing edge. The wing depth distribution, $d(x,y)$, and the cover panel thickness distribution, $t(x,y)$, are expressed in for form of polynomials. These polynomials are chosen so that a realistic and general distribution can be expressed with a minimum number of constants (the design variables, of which there are 6) and also $d(x,y)$ must satisfy the condition of zero thickness at the leading and the trailing edges. Prespecified portions of the chord length from the leading and the trailing edges are excluded as nonstructural in computing the stiffness characteristics of the wing structures. A Ritz type displacement method is used to formulate the stiffness characteristics including transverse shear deformation. Sets of polynomials are determined so that each function is strongly independent from the others in the set. The structural mass densities of the core and the cover plate materials are considered to be uniform. Fuel stored in the wing is also considered to be distributed uniformly in the core and the total amount of fuel in the wing can be designated arbitrarily. Large concentrated masses such as engines are included as point masses at specified locations in the neutral plane of the wing. The well-known[10] procedures for assembling the so-called "consistent mass matrix" are used. Second-order piston theory,[11] in which the wing depth distribution is taken into account, is used to predict both the steady and unsteady pressure distributions on the wing surfaces.

The finite element method is used in the Phase II capability to represent the structural characteristics of the

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wing. An idealization consisting of a combination of triangular membrane elements (cover panels), rectangular shear panels (webs) and pin-jointed bars (stringer or rib flanges) was used.

Tuning masses are included for the purpose of suppressing flutter and have no load carrying capability as structural members. These problems are formulated as mathematical programing problems in the general form: Minimize $f(\mathbf{X})$ subject to $g_j(\mathbf{X}) \leq 0, j = 1, 2, \ldots, NC$ where $\mathbf{X}$ is the vector of design variables, $f(\mathbf{X})$ is the objective function, and the $g_j(\mathbf{X})$ are constraint functions.

Efficiency of the design procedure is of critical importance in problems of this scale and hence a next level of idealization or approximation is useful to expedite the analysis.

Behavior analysis and design; phase I: A complete equation of motion for the model can be written in a matrix form and reduced to a smaller system of equations through the elimination of coordinates not associated with inertial forces. Modification of the reduced stiffness matrix by the aerodynamic stiffness matrices eliminates the necessity of iteration of the displacement state and the pressure distribution on the wing surface. Determination of the root angle of attack to provide specified lift requires an iteration process. The natural vibration frequencies are computed by solving a standard eigenvalue problem using Householder's method. The flutter condition is found as the least Mach number solution of a nonlinear, complex, algebraic equation, with respect to two unknowns. This is obtained in the Phase I capability by directly minimizing the absolute value of the flutter determinant by means of the conjugate gradient method.[12] The aerodynamic drag acting on the wing may be separated into two parts. Pressure drag can be computed by integrating the drag component of the local pressure over the entire wing surface. Friction drag is computed by integrating the drag component of the friction stress over the entire wing surface.

The design problem can be placed in the form given above. The objective function for Phase I is chosen as a function of the structural weight of the wing and the aerodynamic drag. The behavior constraints are considered for every combination of the flight and fuel conditions and impose limits on stress, displacement, root angle of attack, gross lift, natural frequencies, flutter speed, plan form parameters, wing area, wing depth minimum and minimum gauge.

Zoutendijk's method of feasible directions (See Ref. 13 or for computational detail Ref. 14) was selected for Phase I. Some useful modifications were made to the basic method.

Behavior analysis and design; phase II: The stresses, displacements, natural vibration frequencies and flutter conditions are to be computed. The governing equations for these quantities are obtainable through standard procedures. The number of degrees of freedom involved in the static analysis is reduced to one half by assuming the wing to be symmetric about its middle plane.

Frequencies of natural vibration in a vacuum are computed by solving an eigenvalue problem and it is desirable to further reduce the degrees of freedom of the system. The reduction technique used in the present study is similar to the one outlined in Ref. 15.

The flutter mode can be expressed by: $\mathbf{Y} = U\xi$, where the columns of the matrix $\mathbf{U}$ represent the s lowest natural vibration modes and ξ is a vector of modal participation coefficients. The two unknown parameters are ω and V and the combination having the lowest V that makes the determinant vanish is the flutter condition. This problem is solved by a double iteration process given in detail in Ref. 11. The derivatives with respect to the design variables of the static responses are computed by finite difference and those for natural frequencies and Mach number are computed using analytical relations obtained by assuming that the mode shapes are independent of small changes in the design variables. The objective function used in Phase II is the structural weight of the wing. Multiple flight conditions and an arbitrary fuel condition and gross lift requirement can be specified. Constraints are imposed on essentially the same items as in Phase I except for those relating to shape, which are not present in Phase II, and some added ones on element details. In Phase II, the sequential unconstrained minimization technique (SUMT) is used by transforming the constrained problem into an unconstrained problem through an interior penalty function formulation.

In both phases of design, computation time required for one complete design ranged from 0.6 to 1.5 hr on a UNIVAC 1108 computer. This was achieved by taking advantage of special structures of the problems and by utilizing human judgement at various stages during run time.

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